Sound Field Analysis of Monumental Structures by the Application of Diffusion Equation Model

Zühre Sü Gül[*1], Ning Xiang[*2], and Mehmet Çalışkan[3]
[1]Middle East Technical University, Department of Architecture; MEZZO Stüdyo Ltd [2]Graduate program of architectural acoustics, Rensselaer Polytechnic Institute, [3]Middle East Technical University, Department of Mechanical Engineering
*Corresponding author: ODTU-Technopolis, Galyum Blok:No 21-A, 06800, Ankara, Turkey, zuhre@mezzostudyo.com

Abstract: Sound energy distribution patterns within enclosed spaces are the basic concerns of architectural acoustics. Energy decays are analyzed for major acoustical parameter estimations, while spatial energy distribution and flow vectors are indicative in the analysis of sound energy circulation and concentration zones. In this study the acoustical field of a real-size multi-domed monumental structure is analyzed using a diffusion equation model (DEM) as part of a larger research project. For applying the DEM over the solid model of the superstructure, basic PDE module of COMSOL is utilized. The time dependent solution is analyzed via COMSOL slice, volume and arrow volume (flow vector) plot groups. Results highlighted specific energy accumulation zones inside the space indicating probable reasons for non-exponential sound energy decay formation. This study emphasizes finite element modeling by DEM application to be considered as a practical and scientific method of room acoustics predictions, particularly for in-depth sound field analysis.

Keywords: Diffusion equation model, acoustical coupling

1. Introduction

This paper presents the implementation of a `diffusion equation modeling' and its findings within the context of a larger research project in relation to architectural acoustics and acoustical coupling. The major research question is whether single-volume systems with particular architectural compositions could provide the circumstances for the formation of non-exponential sound energy decays inside that specific enclosure. Within that respect, multiple dome superstructures are identified to be the sample group for computational models and/or real size experiments on acoustical data collection and analysis.

The pilot case is Süleymaniye Mosque in İstanbul (1550-1157). The major architectural features that dominate the silhouette of the superstructure are the multiple-dome upper structure, flying buttresses, arches and elephant feet. The central dome is supported on two sides by semi domes. Side aisles are sheltered by five smaller domes. The inner plan of the mosque is a rectangle measuring 63 by 69 m. Main dome has a diameter of 26 m and the height of the dome from the ground to the keystone is 48 m[1]. The Mosque has an approximate volume of 90,000 m^3. Basic interior materials are stone and plaster-paint for the upper structure and carpet for floors. The particular sound field generated within this single exceptionally large but physically fragmented volume within multi-dome upper shell formation is investigated.

Previous research on the pilot case incorporates room-acoustic computer simulations, real-sized field measurements and data analysis -decay parameter estimations-[2,3]. Preliminary findings necessitated further scientific understanding of the non-exponential energy decay formation within such single volume geometry. At this point, the diffusion equation model (DEM) is applied for investigating spatial variations within the sound field of superstructure due to different volumetric couplings.

2. Diffusion Equation Model (DEM)

The DEM theory for room acoustics applications have recently been investigated by many researchers[4,5,6] and proved further advantageous in compare to statistical theory, wave theory and geometrical acoustics approaches. A COMSOL based DEM solution is superior to ray-tracing simulations in respect to its high computational speed, and additional

outputs of spatial energy and flow vector analysis.

In application of DEM analysis to the pilot case initially the solid geometry of the superstructure, relying on the latest field drawings[7], is generated using AutoCAD. The model is imported in COMSOL Multi-physics v4.03. The basic coefficient form PDE module is utilized in defining domain and boundary equations of the DEM as explained in the following.

2.1. Interior Diffusion Equation

In the presence of an omni-directional sound source within a room or region/domain (V) with time-dependent energy density $q(r,t)$, the particle density or the acoustic energy density (w) at a position (r) and time (t) is;

$$\frac{\partial w(r,t)}{\partial t} - D\nabla^2 w(r,t) + cmw(r,t) = q(r,t), \quad \in V$$
(1)

where, ∇^2 is the Laplace operator, D is the 'diffusion coefficient', c is the speed of sound and m is the coefficient of air attenuation. Diffusion coefficient D in eq. (1) is a term that takes into account the room morphology through its mean free path which is;

$$D = \frac{\lambda c}{3} = \frac{4Vc}{3S}$$
(2)

where, λ is the mean free path, c is the speed of sound, V is the volume of the room and S is the total surface area of the room. In eq. (1) the source term $q(r,t)$ is zero for any subdomain where no source is present. In the theory of the diffusion of particles by scatterers, a point source emitting q particles per second is modelled by a source term equal to $q\delta(r - r_s)$, where r_s denotes the position of the source. Similarly, in a room-acoustics problem, for time-dependent solutions a point source with an arbitrary acoustic power of $P(t)$ can be modelled as follows;

$$q(r_s,t) = P(t)\delta(r - r_s),$$
(3)

where, δ is the Dirac-delta function.[4,6]

Equation (3) is defined as a point source within coefficient form PDE, in given

coordinates (*imam* at *mihrab* position). Step function is used for providing gradual/continuous decrease on the source power from initiation to cut off within a very short time span (0.1s).

2.2. Boundary Conditions

The boundary condition that is established to include the energy exchanges on enclosing surfaces is expressed as follows;

$$J(r,t) \cdot n = -Dw(r,t) \cdot n = A_X cw(r,t), \quad on\,S$$
(4)

where, c is the speed of sound, A_X is an exchange coefficient or the so called *absorption factor*. For implementation over the case Mosque, given the fact that the room has an absorptive carpet floor -for specific octave bands- versus a low absorptive/reflective upper shell, the boundary condition fits best to the modified mixed boundary model. For the mixed boundary conditions the absorption factor is defined as follows[5];

$$A_X = A_M = \frac{\alpha}{2(2 - \alpha)}$$
(5)

Combining eq. (4) and eq. (5) gives the resulting system of the boundary equation, as follows;

$$-D\frac{\partial w(r,t)}{\partial n} = \frac{c\alpha}{2(2 - \alpha)}w(r,t), \quad on\,S$$
(6)

Sound absorption coefficient information for the upper shell structure (stone-paint) and the floor (carpet) are implemented in boundary/ absorption term under flux/source, for manually selected boundaries. The mean free path of the room is estimated to be 18.26 m, and accordingly the mean free time of the room is 0.053 s. Maximum mesh element sizes are selected to be much smaller than the mean free path.

Eqs. (1), (3) and (6) are solved for total of 124,788 linear Lagrange-type mesh elements (Figure 1). In a time-dependent solution, resulting $w(r,t)$'s after relevant logarithmic scaling, are used for spatial sound energy density distribution and sound energy flow vector analysis.
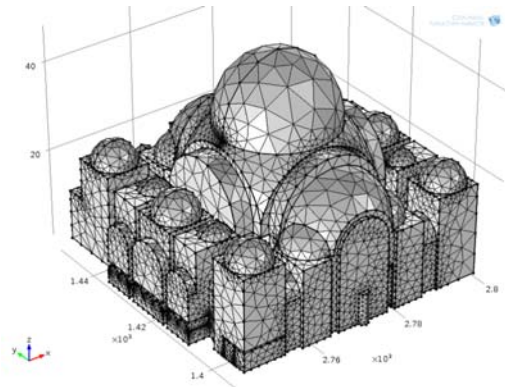
Figure 11. Süleymaniye Mosque solid mesh model

3. Results and Discussion

For visualizing spatial sound energy level distributions, the DEM solution of the pilot study is presented for 250 Hz and 1000 Hz as sample octave bands. The source is in front of *mihrab* wall (standing *imam* position); a representative case for common mosque prayer use. In Figure 2 volume and slice plots are given for 1 kHz at 0.1s indicating the ignition of the sound source and the direct sound effect. In Figure 3, the same conditions are given for time 2 s, indicating a sample period after steady-state condition, where the reverberation tail is in drop. All time intervals closer before and after this time shows a similar trend. Two dimensional mapping of impulse-response derived sound-energy flow vectors are illustrated over section through the *mihrab* wall for 250 Hz, for initial time decay periods (Figure 4).
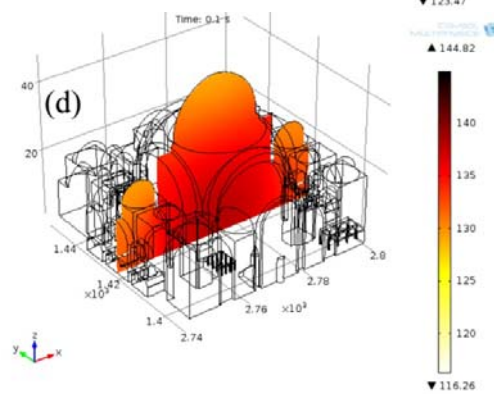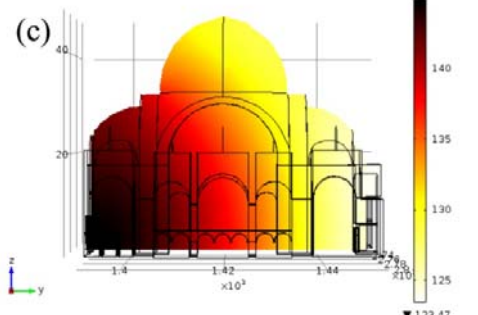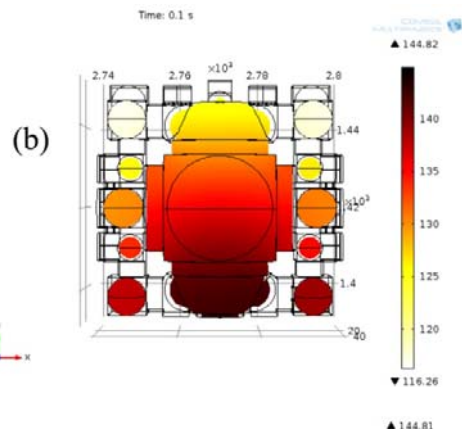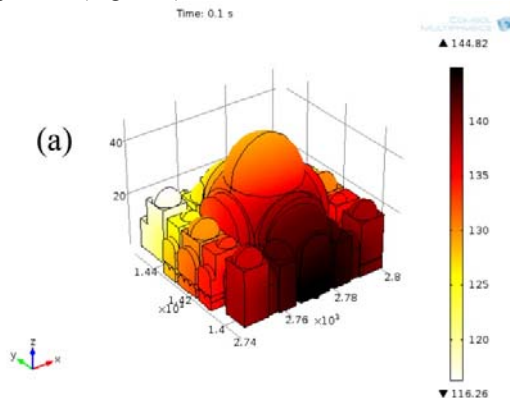

Figure 22. Spatial sound energy level (dB) distribution; volume and slice plots of Süleymaniye Mosque DEM solution, for 1 kHz, time: 0.1 s. a) axonometric view, b) plan view, c) section through the mihrab wall, central axis, d) section parallel to the mihrab wall, central axis

As observed in Figure 2 the concentration of sound energy density is at the front part of the *mihrab* wall, where the point source is defined. The energy starts to flow from *mihrab* wall towards the back of prayer hall, while, at this point the central dome and back wall aisles has not been completely filled with the sound

energy. The inverse section (Figure 2d) indicates a more even distribution and an average sound level. The zones closer to the floor (receiver/prayer heights) underneath the central dome at that direct sound period get more energy in compare to prayer locations in front of the back wall and get least underneath back wall corner domes.
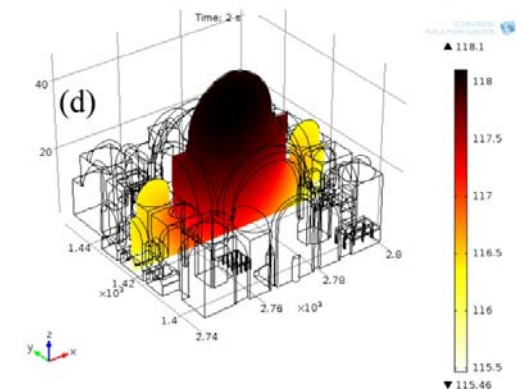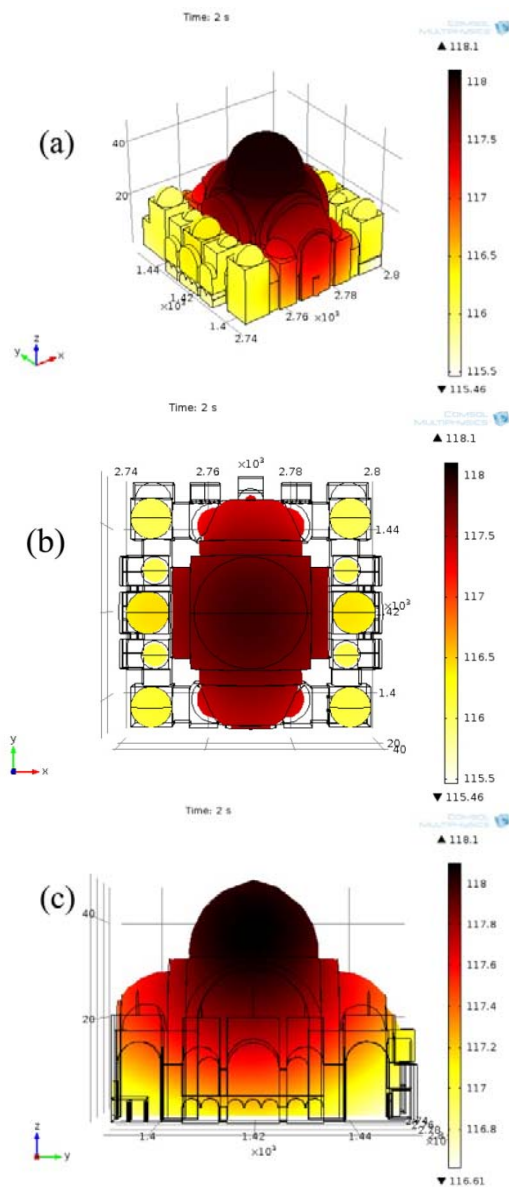

Figure 33. Spatial sound energy level (dB) distribution; volume and slice plots of Süleymaniye Mosque DEM solution, for 1 kHz, time: 2 s. a) axonometric view, b) plan view, c) section through the mihrab wall, central axis, d) section parallel to the mihrab wall, central axis

From Figure 3 plots it can be clearly observed that at this state the sound energy is concentrated at the central axis underneath the main dome and the semi-domes. From this point out, the energy center is the central dome, with its comparatively reflective surfaces and focusing geometry. In architectural acoustics terms, it is beneficial that the dome focusing area is completed much above the receiver height. On the other hand, this energy accumulation center keeps feeding energy back to the floor area, as can be seen from following flow vectors (Figure 4). Another point is that, the side aisles underneath the secondary domes gets substantially less energy compared to the *mihrab* wall section (Figure 3c). This energy fragmentation indicates the zone underneath the main and the secondary domes to work as of a reverberation chamber, while the aisles are fairly dead areas that get later energy feedback from the central zone.

Figure 4, involves supportive arrow volume plots of three-dimensional impulse-response derived sound-energy flow vectors, for 250 Hz. In this case the interval between 0.1 s to 2 s is selected for specific time zones. According to the arrow vector analysis the flow return is around 0.7 s. After 1 s, the energy is stabilized at the central upper zone (main dome), and starts feeding back the rest of the mosque's prayer zones.
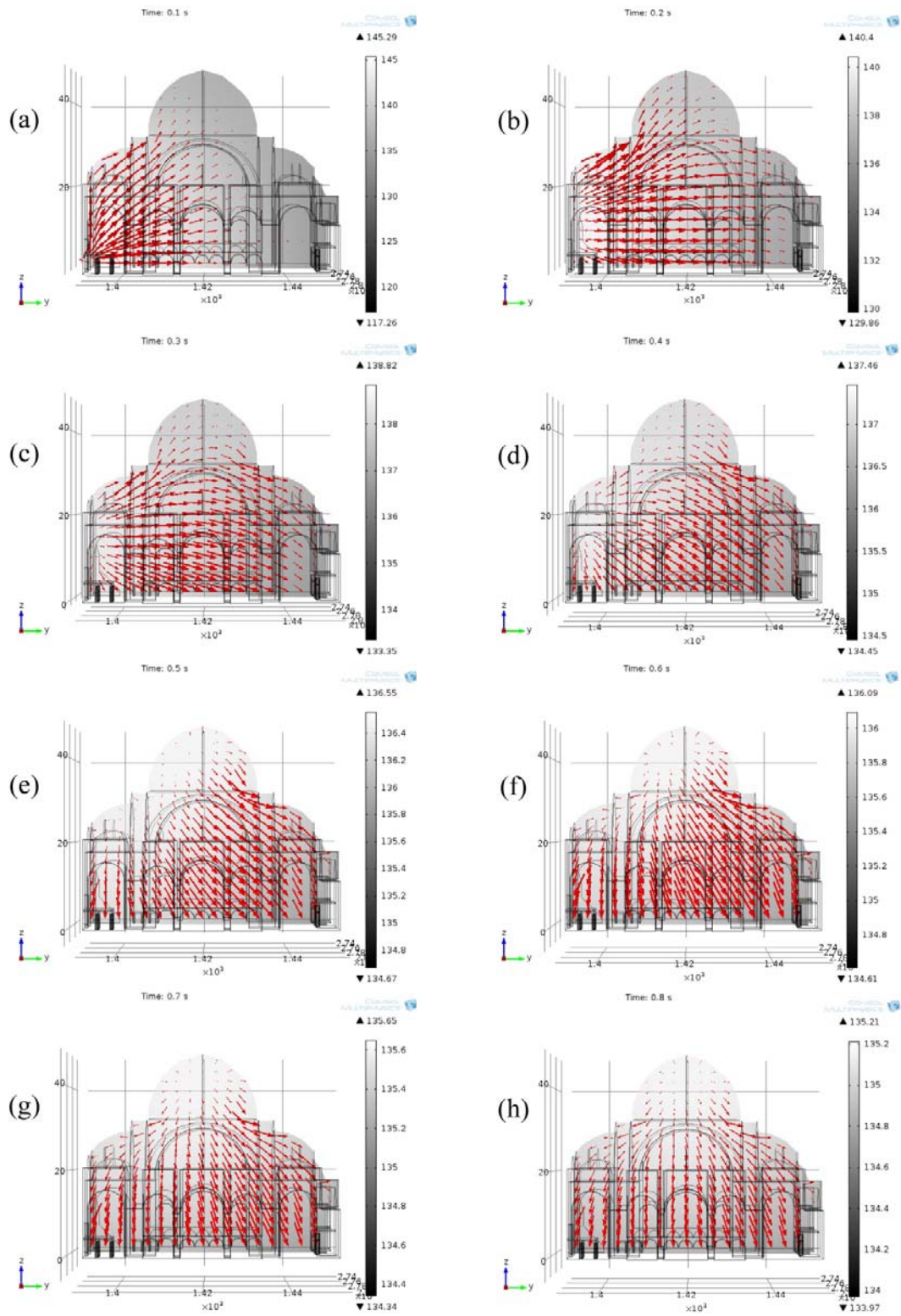
Figure 44. Three-dimensional mapping of impulse-response derived sound-energy flow vectors (arrow volume plots) of Süleymaniye Mosque DEM solution for the time dependent solution, for 250 Hz. a) time: 0.1 s, b) time: 0.2 s, c) time: 0.3 s, d) time: 0.5 s, e) time: 0.7 s, f) time: 1 s, g) time: 2 s, h) time: 4 s.

The frequency is selective in the boundary definition, where the absorptive carpet and reflective upper shell structure -including walls-, together with the dominant geometrical features cause the main energy flow characteristics. The initial part of the decay after the shut off time of the sound source, which is from 0.1 s to 0.8 s in this case, is trivial in terms of many acoustical parameters such as of EDT, C80, D50, LF and ITDG. On the other hand the flow patterns also include some clues on the acoustical coupling trends.

4. Conclusion

In this study the multi-slope decay formation is estimated to be primarily the result of the energy division in between the upper central zone of the mosque -in between the four elephant feet in plan, and approximately where the pendententives of the main dome are started at the section on the boundaries-, and the side aisles underneath the secondary dome structures.

Another reason for the energy divergence within the space is the absorptive and reflective sound area break-up in between carpet and stone and/or plastered brick upper shell and wall surfaces. Specifically for lower octave bands the difference in between sound absorption performance of carpet or plaster are not significantly deviate from each other. For that reason the sound flow is just differ in the manner of raising the energy concentration in the dominating central mihrab wall section by 1 to 2 meters upward the receiver zone in compare to a reflective floor. Therefore, rather than the materials' sound absorption characteristics, the geometrical features and/or architectural divisions in form of central dome, secondary dome and secondary side domes with their characterizing dimensions and heights from the floor together with main axial plan dimensions, are the dominating properties in effect of acoustical coupling formation.

Exercised over an existing monumental structure within the context of this study, the

DEM solution findings are significant in revealing more specific causes of multi-slope decay formation within single volume structures with specific geometric attributes. It should be emphasized that the DEM application by finite element modeling is a practical and scientific method of room acoustics predictions, particularly for in-depth sound field analysis. Thus, COMSOL as a finite element solver could find many grounds in room acoustics applications as in this case and can be utilized over existing structures or over virtual spaces as an acoustical analysis or acoustical design tool.

5. References

1. Mungan, I, Strüktür çözümü, *Bir şaheser Süleymaniye Külliyesi,* Ed. S. Mülayim. Ankara: T.C. Kültür ve Turizm Bakanlığı (2007)
2. Sü Gül, Z., Xiang N. and Çalışkan, M., Investigations on multi-slope sound energy decays in domed structures, *Proceedings of Inter-Noise 2012,* 19-22 August, New York, USA (2012)
3. Sü Gül, Z., Xiang N. and Çalışkan, M., Multiple-slope sound energy decay investigations in single space enclosures with specific geometrical and material attributes, *166th Meeting of the Acoustical Society of America*, San Francisco, California, (2013)
4. Valeau, V., Picaut, J. and Hodgson, M, On the use of a diffusion equation for room-acoustic prediction, *J. Acoust. Soc. Am.*, **119.3**, 1504-1513, (2006)
5. Jing, Y. and Xiang, N., On boundary conditions for the diffusion equation in room acoustic prediction: theory, simulations, and experiments, *J. Acoust. Soc. Am.*, **123.1**, 145-153, (2008)
6. Xiang, N., Jing, Y. and Bockman, A.C., Investigation of acoustically coupled enclosures using a diffusion-equation model, *J. Acoust. Soc. Am.*, **126.3**, 1189-1198, (2009).
7. T.R. Prime Ministry Directorate General of Foundations Archive, *Süleymaniye Camii belgeleri*, Ankara, (2011)